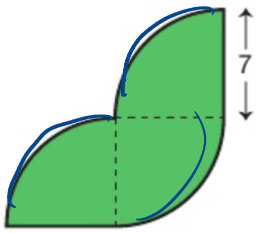
Shape and Space III (1) Circles problem solving

Do now:

Find the perimeter and area of each of the following shapes, giving answers to 3 s.f.
All dimensions are in cm. All arcs are parts of circles.

1 ▶



$A = \frac{3}{4}\pi(7^2) = \frac{147}{4}\pi$

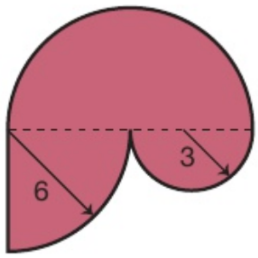
$= 115\text{ cm}^2$ (3sf)

$P = \frac{3}{4}\pi \times 2 \times 7 + 14$

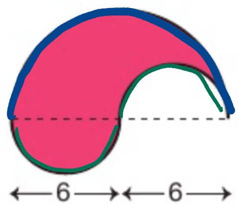
$= \frac{21}{2}\pi + 14$

$= 47.0\text{ cm}$ (3sf)

3 ▶



5 ▶

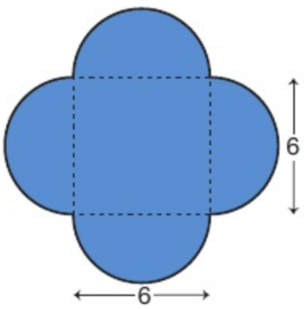


$P = \frac{1}{2}(2\pi \times 6) + 2\pi(3)$

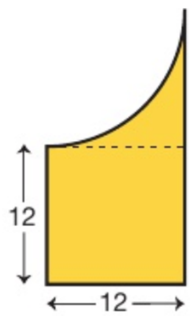
$= 6\pi + 6\pi = 12\pi\text{ cm}$

$A = \frac{1}{2}\pi(6^2) = 18\pi\text{ cm}^2$

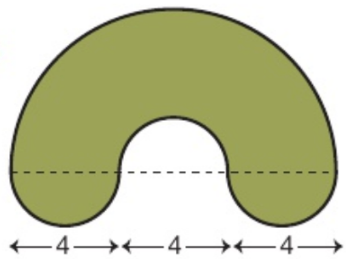
2 ▶



4 ▶



6 ▶



ANSWERS

1 ▶ 47.0 cm, 115 cm²

2 ▶ 37.7 cm, 92.5 cm²

3 ▶ 43.7 cm, 99.0 cm²

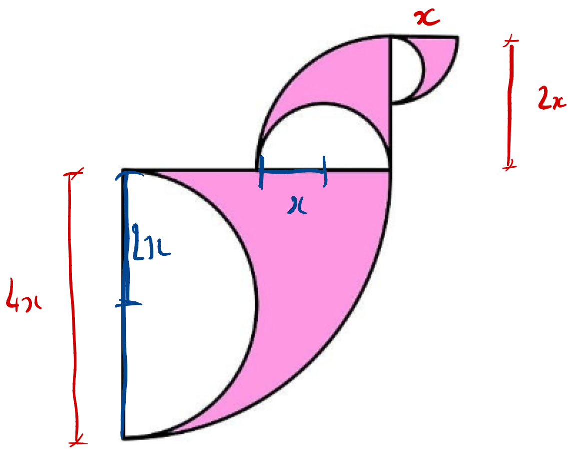
4 ▶ 66.8 cm, 175 cm²

5 ▶ 37.7 cm, 56.5 cm²

6 ▶ 37.7 cm, 62.8 cm²

Extension

1. What fraction of the shape is shaded?



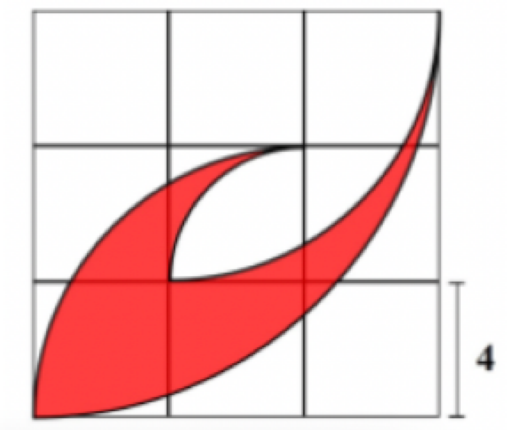
$$\text{TOTAL AREA} = \frac{1}{4}\pi\left[(4x)^2 + (2x)^2 + x^2\right]$$
$$= \frac{1}{4}\pi\left(21x^2\right)$$

$$\text{SHADED AREA} = \frac{1}{4}\pi\left(21x^2\right) - \frac{1}{2}\pi\left[(2x)^2 + x^2 + \left(\tfrac{1}{2}x\right)^2\right]$$
$$= \frac{1}{4}\pi\left(21x^2\right) - \frac{1}{2}\pi\left[\frac{21}{4}x^2\right]$$
$$= \frac{1}{4}\pi\left(21x^2\right) - \frac{1}{4}\pi\left[\frac{21}{2}x^2\right]$$
$$= \frac{1}{4}\pi \times \frac{21}{2}x^2$$

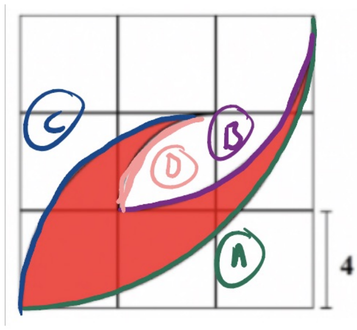
$$\frac{\text{SHADED AREA}}{\text{TOTAL AREA}}$$

$$\text{FRACTION} = \frac{\frac{1}{4}\pi \times \frac{21}{2}x^2}{\frac{1}{4}\pi\left(21x^2\right)} = \frac{1}{2}$$

2. Find the ~~area~~ and perimeter of the shape



Find the perimeter of the shape below



A 

$\frac{1}{4}\pi \times 12 \times 2 = 6\pi$

B 

$\frac{1}{4}\pi \times 8 \times 2 = 4\pi$

C 
IS EQUAL TO B

D 

$\frac{1}{4}\pi \times 4 \times 2 = 2\pi$

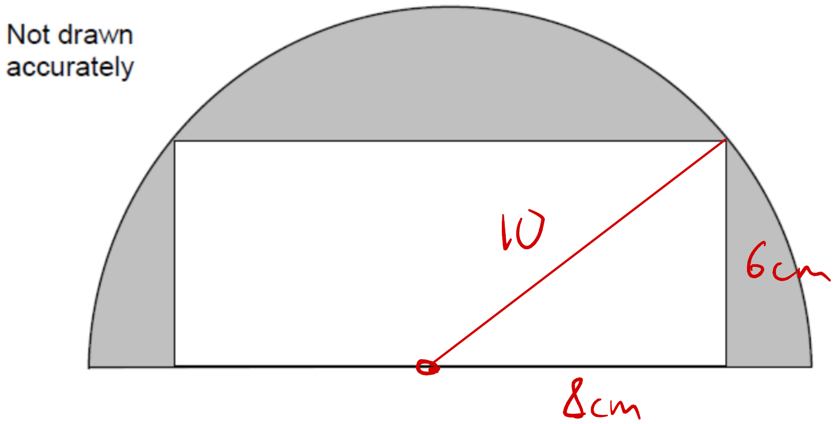
TOTAL PERIMETER $= 16\pi$ UNITS

$= 50.3$ UNITS (1d.p.)

3.

The diagram shows a rectangle inside a semicircle.

$r = 10$



The rectangle has dimensions 16 cm by 6 cm

Work out the shaded area.

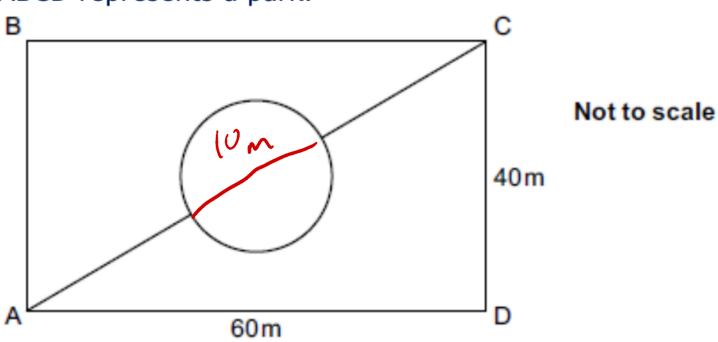
Give your answer in terms of π.

$$\text{SHADED AREA} = \frac{1}{2}\pi(10^2) - (16 \times 6)$$

$$= (50\pi - 96)\ \text{cm}^2$$

4.

The rectangle ABCD represents a park.



The lines show all the paths in the park.

The circular path is in the centre of the rectangle and has a diameter of 10m.

Calculate the shortest distance from A to C across the park, using only the paths shown.

$$AC = \sqrt{60^2 + 40^2} = 20\sqrt{13}$$

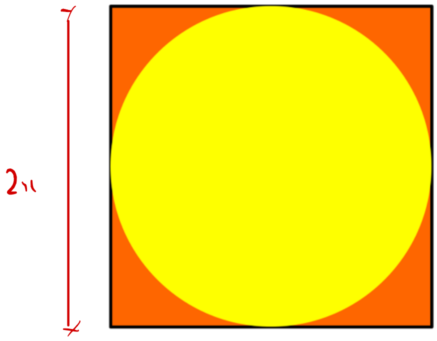
$$\text{AC ON PATHS} = 20\sqrt{13} - 10 + \frac{1}{2}\pi(10)$$

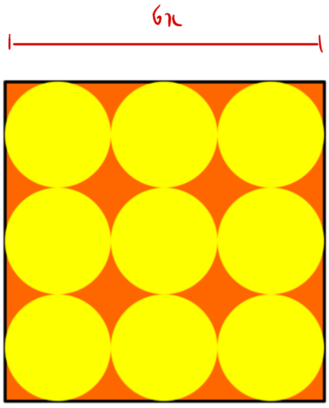
$$= 5\pi + 20\sqrt{13} - 10$$

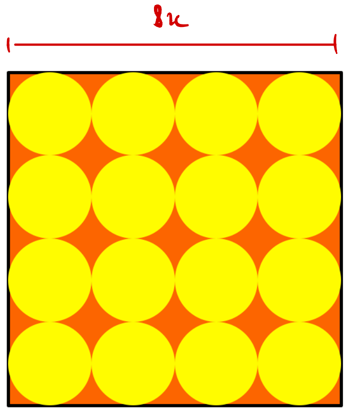
$$= 77.8\text{m (3sf)}$$

TO COMPARE USE FRACTIONS / % $\frac{\text{WASTE}}{\text{TOTAL}}$

Which has more waste? THEY ARE THE SAME







$$\text{WASTE} = (2x)^2 - \pi x^2$$

$$\frac{\text{WASTE}}{\text{TOTAL}} = \frac{4x^2 - \pi x^2}{4x^2} = \frac{4-\pi}{4}$$

$$\text{WASTE} = (6x)^2 - 9(\pi x^2)$$

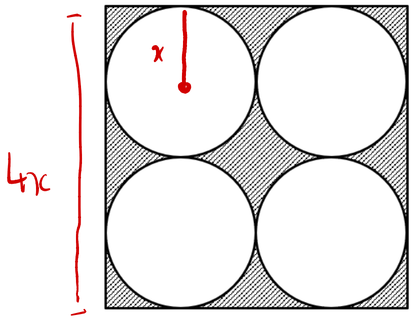
$$\frac{\text{WASTE}}{\text{TOTAL}} = \frac{(36 - 9\pi)\cancel{x^2}}{36\cancel{x^2}} = \frac{4-\pi}{4}$$

$$\text{WASTE} = (8x)^2 - 16(\pi x^2) = 64x^2 - 16\pi x^2 = x^2(64 - 16\pi)$$

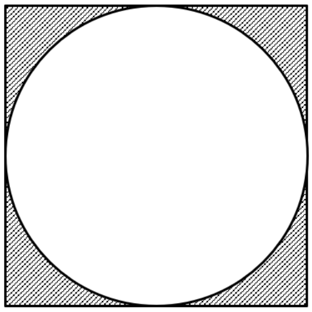
$$\frac{\text{WASTE}}{\text{TOTAL}} = \frac{\cancel{x^2}(64 - 16\pi)}{64\cancel{x^2}} = \frac{64 - 16\pi}{64} = \frac{4-\pi}{4}$$

Which has the greater shaded area? SAME

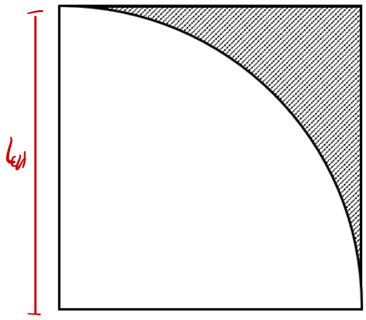
(1)



(2)



(3)



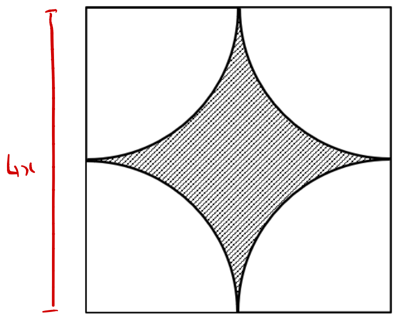
$$\frac{\text{WASTE}}{\text{TOTAL}} = \frac{16x^2 - 4(x)^2\pi}{(4x)^2} = \frac{\cancel{x^2}(16 - 4\pi)}{16\cancel{x^2}} = \frac{4-\pi}{4}$$

SAME AS ABOVE

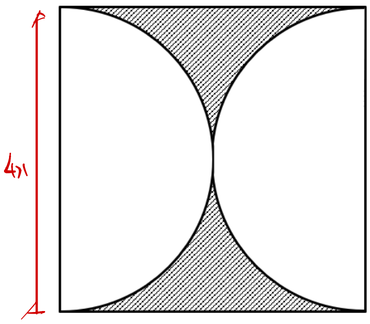
$$\frac{4-\pi}{4}$$

$$\frac{\text{WASTE}}{\text{TOTAL}} = \frac{(4x)^2 - \frac{1}{4}(4x^2)\pi}{16x^2} = \frac{(16 - 4\pi)\cancel{x^2}}{16\cancel{x^2}} = \frac{4-\pi}{4}$$

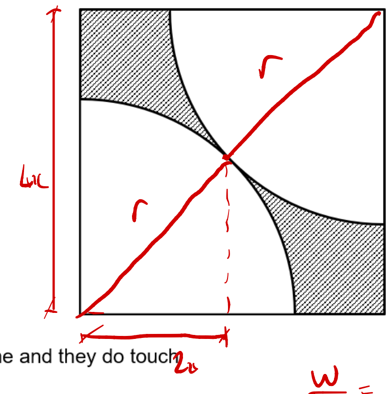
(4)



(5)



(6)



within each of the identical squares, the semicircles and quadrants are the same and they do touch

$$r = \sqrt{(2x)^2 + (2x)^2} = \sqrt{8x^2} = 2\sqrt{2}\,x$$

$$\frac{\text{WASTE}}{\text{TOTAL}} = \frac{16x^2 - \pi(2x)^2}{16x^2} = \frac{\cancel{x^2}(16 - 4\pi)}{16\cancel{x^2}} = \frac{4-\pi}{4}$$

$$\frac{W}{T} = \frac{16x^2 - \pi(2x^2)}{16x^2} = \frac{\cancel{x^2}(16 - 4\pi)}{16\cancel{x^2}} = \frac{4-\pi}{4}$$

$$\frac{W}{T} = \frac{(4x)^2 - \frac{1}{2}\pi(2\sqrt{2}\,x)^2}{16x^2} = \frac{\cancel{x^2}(16 - 4\pi)}{16\cancel{x^2}} = \frac{4-\pi}{4}$$